M.Sc. Second Semester
(Assignment)
MATHEMATICS
FIRST PAPER
Algebra-II

Attempt any two questions.

Q.1 (a) Let V be an n – dimensional vector space over a field F. Let $B = \{b_1, b_2, \dots \dots\ b_n\}$ be a basis of V. Then prove that the dual space V^* has a basis $B^* = \{f_1, f_2, \dots \dots\ f_n\}$ such that $f_i(bi) = \delta_{ij} i, j = 1, 2, \dots, n$ where $\delta_{ij} \in F$ is Kronecker delta.

(b) Let V be a finite dimensional vector space over a field F and W be a subspace of V. Then show that $A\big(A(W)\big) = W$, where $a(W)$ denote annihilator of W.

Q.2 (a) Find the dual basis of the basis $B = \{(1, -1,3),\ (0,1, -1), (0,3, -2)\}$ of $R^3(R)$.

(b) Let V and V' be finite dimensional vector spaces over a field F and $t: V \to V'$ be a linear transformation, then prove that t and t' have the same rank, where t^* is the dual map of t.

Q.3 (a) Prove that a linear transformation $t: V \to V$ is invertible if and only if matrix of t relative to some basis B of V is invertible.

(b) Prove that the similar matrices have the same trace.

Q.4 (a) Let $t: V \to V$ be a linear transformation from a finite dimensional vector space V to itself. Assume that $v_i,\ i = 1,2, \dots \dots n$ are distinct eigen vectors of t corresponding to distinct eigenvalues $\lambda_i,\ i = 1,2, \dots \dots n$ then prove that $\{v_i\}_{i=1}^n$ is a linearly independent set.

(b) Prove that the rank of the product of two matrices never exceed the rank of either matrix.

Q.5 (a) Prove that a square matrix A over a field F is invertible iff $\mathrm{Det}(A) \neq 0$ i.e. A is non singular.

(b) Let A be a square matrix over a field F. Prove that A and A^T have the same characteristic polynomial.

Q.6 (a) If α and β are vector's in an inner product space V then prove that

$$\|\alpha + \beta\| \leq \|\alpha\| + \|\beta\|$$

(b) Let A be an $n \times n$ matrix over a field F .Then prove that a scalar $\lambda \in F$ is an eigen value of A if and only if $\det(A - \lambda I) = 0$

Q.7 If $B = \{\alpha_1, \alpha_2, \dots \dots \alpha_n\}$ is any finite orthonormal set in an inner product space V(F) and if $B \in V(F)$ then prove that:

$\sum_{i=1}^{n} |(\beta, \alpha_i)|^2 \leq \|\beta\|^2$ and

$$\sum_{i=1}^{n} |(\beta, \alpha_i)|^2 = \|\beta\|^2 \Leftrightarrow \beta \in L(B)$$

Q.8 (a) Write four properties of adjoint of a linear transformation between finite dimensional inner product spaces.

(b) State and prove that Principal Axis Theorem.

M.Sc. Second Semester

(Assignment)

MATHEMATICS

SECOND PAPER

Topology

Attempt any two questions.

Q.1 (a) Let X is an infinite set and let τ be the family consisting empty set Φ and all those non – empty subsets of X whose complements are finite. Then prove that (x,τ) is a topological space.

(b) Let (X,τ) be a topological space and let A, B be any two subsets of X. If A' is the collection of all limit points of A, then prove that

i) $A \subseteq B \implies A' \subseteq B'$

ii) $(A \cup B)' = A' \cup B'$

iii) $(A \cap B)' \subseteq A' \cap B'$

Q.2 (a) Let (X,τ) be a topological space and let $A \subseteq X$. Define interior point of A. If Å is the collection of all interior points of A, then prove that Å is the largest open set contained in A.

(b) Define closure of a subset A of a topological space (X,τ). Let $X = \{a,b,c\}$ and let $\tau = \{\Phi, X, \{a\}, \{b\}, \{a,b\}\}$ be a topology on X. Find the closure of sets $\{b\}$ and $\{b,c\}$

Q.3. (a) Define Continuous Mapping. If X and Y are two topological spaces, then prove that a mapping $f: X \to Y$ is continuous if and only if $f^{-1}(B^0) \subseteq \{f^{-1}(B)\}^0$ for every subset B of Y. B^0 denotes interior of set B.

(b) Define filter and ultrafilter on a non – empty set X. Prove that every filter on X is contained in an ultrafilter on X.

Q.4. (a) Let (X,τ_1) and (Y,τ_2) be two topological spaces. Then prove that a mapping $f: X \to Y$ is closed if and only if $\overline{f\ (A)} \subseteq f\ (\bar{A})$ for every subset A of X ($\bar{A}$ denotes closure of A).

(b) Prove that a topological space (X,τ) is a Hausdorff space if and only if every net in X can Converge to at most one point.

Q.5 (a) Let f and g be a continuous functions on a topological space (X, τ_1) into a Hausdorff space (T_2 −space) (Y, τ_2). Then prove that the set $\{x \in X : f(x) = g(x)\}$ is a closed subset of X.

(b) Prove that every open and continuous image of a locally compact topological space is locally compact.

Q.6 (a) Prove that a topological space (x, τ) is normal if and only if for any closed subset F of X and an open set G containing F, there exists an open set V such that $F \subseteq V \subseteq \bar{V} \subseteq G$.
(b) Prove that every closed subset of a compact topological space is compact.

Q.7 (a) Prove that the product space of two T_1 spaces is a T_1 - space.
(b**)** Prove that a topological space (X,T) is disconnected if and only if there exists a continuous mapping of X onto the two points discrete space $(\{0,1\}, D)$.

Q.8 (a) Define Quotient Topology. Let f be a continuous mapping of a topological space X onto a topological space Y and let Y have the quotient topology. Then, prove that a mapping g from Y onto a topological space Z is continuous if and only if the composite mapping gof is continuous.
(b) Define locally connected topological spaces with one example. Prove that every open subspace of a locally connected topological space is locally connected.

M.Sc. Second Semester

(Assignment)

MATHEMATICS

THIRD PAPER

Differential Equations-II

Attempt any two questions.

Q.1 (a) Reduce the equation

$$(n-1)^2\frac{\partial^2 z}{\partial x^2} - y^{2n}\frac{\partial^2 z}{\partial y^2} = ny^{2n-1}\frac{\partial z}{\partial y}$$

to canonical form and find its general solution.

(b) Classify the equations:

(i) $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 2\frac{\partial^2 u}{\partial x\partial y} + 2\frac{\partial^2 u}{\partial y\partial z}$

(ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{C^2}\frac{\partial^2 u}{\partial t^2}$

Q.2 (a) Classify the equation:

$$\frac{\partial^2 u}{\partial x^2} + 3\frac{\partial^2 u}{\partial y^2} + 84\frac{\partial^2 u}{\partial z^2} + 28\frac{\partial^2 u}{\partial y\partial z} + 16\frac{\partial^2 u}{\partial z\partial x} + 2\frac{\partial^2 u}{\partial x\partial y} = 0$$

(b) Reduce the equation

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x\partial y} + \frac{\partial^2 z}{\partial y^2} = 0$$

to canonical form and hence solve it.

Q.3 (a) Find the eigen values and the corresponding eigenfunctions of the boundary value problem: $y'' + \lambda y = 0$, $y(0) = 0$, $y(1) = 0$

(b) Find the solution of Sturm – Liouville problem:

$$y'' + \frac{1}{x}y' + \frac{\lambda}{x^2}y = 0, \quad 1 \le x \le 2$$

with boundary conditions

$y(1) = 0,\ y(2) = 0$

Q.4 (a) Find the eigenvalues and eigen functions of the given boundary value problems. Assume that all eigen values are real:

$y'' + \lambda y = 0$

(i) $y'(0) = 0,\ y(\Pi) = 0$

(ii) $y'(0) = 0,\ y'(1) = 0$

(b) Prove that the eigen values of Sturm – Liouville system are real.

Q.5 (a) Obtain the solution of the non-homogeneous Sturm-Liouville boundary value problem: $y'' + y = f(x);\ y(0) = 0,\ y'(\Pi) = 0$

where

$$f(x) = \begin{cases} 2x/\Pi & , 0 \le x \le \Pi/2 \\ 1 & , \Pi/2 \le x \le \Pi \end{cases}$$

(b) Find the most general function $X(x)$ and $Y(y)$, each of one variable, such that $z(x, y) = XY$ satisfies the partial differential equation:

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial y}$$

obtain a solution of the above equation which satisfies the boundary condition:

$z = 0$ when $x = 0$ or Π

$z = \sin 3x$ when $y = 0$ and $0 < x < \Pi$

Q.6 (a) Find the solution of the non-homogeneous boundary value problem:

$y'' + 6y = f(x)\ ,\ y(0) = 0,\ 3y(\Pi) - y'(\Pi) = 0$

by the method of eigen function expansion.

(b) Solve the Heat Conduction Equation: $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$

given that $u = 0$ when $t \to \infty\ ,\ x = 0, x = l$

Q.7 (a) Find the appropriate Green's Function for the equation:

$$\frac{d^2y}{dx^2} + \frac{1}{4}y = f(x)$$

with the boundary conditions $y(0) = 0$ and $y(\Pi) = 0$. Hence, solve for $f(x) = x/2$

(b) Solve the boundary value problem:

$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x,\ \ y(1) = 0\ ,\ y(2) = 0$ using Green's Function.

Q.8 (a) Find the Green's function for the equation:

$$\frac{d^2y}{dx^2} - y = f(x),\ 0 \le x \le 1$$

with the boundary conditions $y(0) = 0, y'(1) = 0$

(b) Find the modified Green's Function for the boundary value problem:

$y'' = x - L/2$

with the boundary conditions $y'(0) = 0\ ,\ y'(L) = 0$

M.Sc. Second Semester

(Assignment)

MATHEMATICS

FOURTH PAPER

Riemannian Geometry & Tensor Analysis

Attempt any two questions.

Q.1. (a) Prove that the necessary and sufficient condition that every helix on a cylinder is a geodesic.

(b) A particle is constrained to move on a smooth surface under no force except the normal reaction. Prove that its path is a geodesic.

Q.2 (a) If K and T are curvature and torsion of a geodesic then prove that:

$T^2 = (k - k_a)(k_b - k)$

(b) If the parametric curves $u =$ constant and $v =$ constant, are orthogonal ,then show that their geodesic curvatures are

$\frac{1}{\sqrt{GE}} \frac{\partial}{\partial u} \sqrt{G}$ and $-\frac{1}{\sqrt{GE}} \frac{\partial}{\partial u} \sqrt{E}$

Q.3. If a contravariant vector has components $\dot{x}, \dot{y}$ in rectangular Cartesian coordinates then $\dot{r}, \dot{\theta}$ are components in polar coordinates and if a vector has components $\ddot{x}, \ddot{y}$ in Cartesian coordinates then prove that they are $\ddot{r} - r\dot{\theta}^2$ and $\ddot{\theta} + \frac{2}{r}\dot{r}\dot{\theta}$ in polar coordinates where dots represent differentiation w.r.t. parameter t.

Q.4. State and prove Quotient Law of Tensors.

Q.5 Prove that

$$A^j_{i;j} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j} \left(A^j_i, \sqrt{g}\right) - A^j_k \begin{Bmatrix} k \\ ij \end{Bmatrix}$$

Show also that if the associate tensor of A^{ij} is symmetric then

$$A^j_{i;j} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j} \left(A^j_i, \sqrt{g}\right) - \frac{1}{2} A^{jk} \frac{\partial}{\partial x^i} g_{jk}$$

where (j) indicates covariant differentiation.

Q.6 (a) If A_{ij} is a skew symmetric tensor of rank two then show that

$$\frac{\partial A_{ij}}{\partial x^k} + \frac{\partial A_{jk}}{\partial x^i} + \frac{\partial A_{ki}}{\partial x^j}$$

is a tensor.

(b) Prove that

$$\left\{ \begin{matrix} i \\ ij \end{matrix} \right\} = \frac{1}{2g} \frac{\partial g}{\partial x^j}$$

Q.7 The metric of V_2 formed by the surface of sphere of radius r is given by

$$ds^2 = r^2 d\theta^2 + r^2 sin^2\theta \, d\phi^2$$

in a spherical polar coordinates then show that the surface of sphere is a surface of constant curvature $\frac{1}{r^2}$.

Q.8. If $ds^2 = e^{2\theta}dx^2 + dy^2 + dz^2 + e^{2\phi}dt^2$

where θ and ϕ are functions of $x -$ only then prove that Riemann – Christoffel tensor vanishes if

$$\phi'' - \theta'\phi' + {\phi'}^2 = 0$$

where a prime denotes differentiation w.r.t x. Also if $\theta + \phi =$ constant then prove that space time is flat provided that

$$\phi = \log(ax + b)^{1/2}$$

M.Sc. Second Semester

MATHEMATICS

FIFTH PAPER

Hydrodynamics

Attempt any two questions.

1. (a) Derive the equation of continuity in cylindrical polar coordinates.

(b) Show that the variable ellipsoid $\frac{x^2}{a^2k^2t^4} + kt^2\left(\frac{y^2}{b^2} + \frac{z^2}{c^2}\right) = 1$ is a possible form of the boundary surface of a liquid at time t.

2. (a) If the velocity of an incompressible fluid flow at the point (x, y, z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2-r^2}{r^5}\right)$, then prove that the liquid motion is possible and the velocity potential is $\frac{\cos\theta}{r^2}$.

(b) The particles of a fluid move symmetrically in space with regard to a fixed centre, prove that the equation of continuity is $\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \frac{\rho}{r^2}\frac{\partial(r^2u)}{\partial r} = 0$, where u is the velocity at a distance r.

3. (a) State and prove Bernoulli's Theorem.

(b) Liquid is contained between two parallel planes; the free surface is a circular cylinder of radius 'a' whose axis is perpendicular to the planes. All the liquid within a concentric circular cylinder of radius 'b' is suddenly annihilated. Prove that if 'Π' be the pressure at the outer surface, the initial pressure at any point of the liquid, distance r from the centre is $\Pi.\frac{\log r - \log b}{\log a - \log b}$.

4. Derive all the three equations of Cauchy's integrals.

5. (a) Define Images in two dimensions. Determine the image of the source with respect to the Straight Line

(b) Two sources each of strength m are placed at the points $(-a, 0), (a, 0)$ and a sink of strength$-2m$ at the origin. Show that the stream lines are the curves $(x^2 + y^2)^2 = a^2(x^2 - y^2 + \lambda xy)$, where λ is a variable parameter. Show also that the fluid speed at any point is $\frac{2ma^2}{r_1r_2r_3}$, where $r_{1,}r_2$andr_3 are the distances of the point from the sources and the sink.

6. (a) Show that in case of an irrotational flow the two stream lines cut at right angles at the stagnation point.

(b) Give and demonstrate physical meaning of the value of a stream function at a point.

7. (a) Prove that the product of cross section and vorticity at any point on a vortex filament is constant along the filament end for all time when the body forces are conservative and the pressure is a single valued function of density only.

(b) Find the necessary and sufficient condition that the vortex line maybe at right angles to the steam lines

8. (a) Prove that the image system of a vortex of the strength k situated at the point outside a circular cylinder consists of a vortex of strength -k at the inverse point and a vortex of strength k at the centre.

(b) Incompressible fluid the vorticity at every point is constant in magnitude and direction. Show that the components of velocity u, v, w are solutions of Laplace's equation.

M.Sc. Second Semester
(Assignment)
MATHEMATICS
SIXTH PAPER
Special Function-II

Attempt any two questions.

Q.1 (a) State and prove generating function for $J_n(x)$.

(b) Prove that

$$2n\, J_n(x) = x[\, J_{n-1}(x) + J_{n+1}(x)]$$

Q.2 (a) Prove that

$$\int_0^t [x(t-x)]^{-\frac{1}{2}} \exp[4x(t-x)]\, dx = \pi \exp\left(\frac{1}{2}\, t^2\right)\, I_0\, \left(\frac{1}{2} t^2\right)$$

(b) Prove that

$$\frac{d}{dx}\left[\frac{J_{-n}(x)}{J_n(x)}\right] = (-)\frac{2\, Sin\, n\, \pi}{\pi\, x\, {J_n}^2\, (x)}$$

Q.3. (a) Prove that

$$H_n(x) = (-1)^n \exp(x^2)\frac{d^n}{d_x n}\, \{\exp(-x^2)\}$$

and find the value of $\int_0^x \bar{e}^y\;\; H_n(y)dy$

(b) Prove that

$$\int_{-\infty}^{\infty} x^2\, \bar{e}^x \{H_n(x)\}^2\, dx = \sqrt{\pi}\; 2^n\, n!\left(n + \frac{1}{2}\right)$$

Q.4 (a) Prove that

$$(n+1)L_{n+1}(x) = (2n+1-x) \quad L_n(x) - x\, L_{n-1}(x)$$

(b) Prove that

$$\int_0^1 x^{\propto}(t-x)^{\beta-1} \quad {}_n{}^{(\propto)}(x)dx = \frac{\Gamma(1+\propto)\Gamma(\beta)(1+\propto)_n t^{\propto+\beta}}{\Gamma(1+\propto+\beta)\; (1+\propto+\beta)_n} \quad {}_n{}^{(\propto+\beta)}(x).$$

Q.5. (a) Prove that

$$\int_{-1}^{1}(1-x)^{\propto}(1+x)^{\beta}\;\; {P_n}^{(\propto,\beta)}(x){P_m}^{(\propto,\beta)}(x)\; dx = 0, m \neq n$$

where $Re(\propto) > -1$ and $Re(\beta) > -1$.

(b) Prove that

$$\sum_{n=0}^{\infty} \frac{(1+x)^n\, P_n^{(\propto,\beta)}\left(\frac{1-x}{1+x}\right)\, t^n}{(1+\propto)_n\,(1+\beta)_n} = {}_0F_1\,[-\,;1+\propto;-xt]\;\; {}_0F_1[-\,;1+\beta\,;t]$$

Q.6 (a) Prove that $(x-1)\frac{d}{dx}\,P_n^{(\propto,\beta)}(x) = n\,P_n^{(\propto,\beta)}(x) - (\propto+n)P_{n-1}^{(\propto,\beta)}(x)$

(b) Prove that

$$\int_{-1}^{1}(1-x)^{\propto}\;P_n^{(\propto,\beta)}(x)\;\;P_k^{(\propto,0)}(x)\,dx$$

$$= \frac{2^{1+\propto}\,(-1)^{n-k}(\beta)_{n-k}(1+\propto+\beta+n)_k}{(n-k)!\,(1+\propto+n)_{k+1}}$$

Q.7 (a) Prove that $T_n(x)$ and $U_n(x)$ are independent solutions of Chebyshev's differential equation.

(b) Prove that

$$(1-x^2)\,T_n'\,(x) = -n\,x\,T_n(x) + n\,T_{n-1}(x)$$

Q.8 (a) State and prove generating function for Chebyshev's polynomials first kind.

(b) Prove that

$$\frac{1}{\sqrt{(1-x)^2}}\;U_n(x)$$

satisfies the differential equation:

$$(1-x^2)\,\frac{d^2u}{dx^2} - 3x\,\frac{du}{dx} + (n^2-1)\,u = 0.$$

M.Sc. Fourth Semester

(Assignment)

MATHEMATICS

FIRST PAPER

Functional Analysis and Advanced Calculus

Attempt any two questions.

Q.1. (a) State & prove Existence Theorem.

(b) Prove that an operator T on Hilbert space H is self – adjoint if and only if (Tx, x) is real for all x in H.

Q.2. (a) If P and Q are the projections on closed linear subspaces M and N of a Hilbert space H, then show that $M \perp N$ if and only if PQ = O if and only if QP = O.

(b) If P_1, P_2 ……. P_n are the projection on closed linear subspaces M_1, M_2 ……. M_n of a Hilbert space H then prove that $P = P_1+P_2+\ldots\ldots\ldots+P_n$ is a projection if and if the Pi's are pair wise orthogonal and P is projection on $M = M_1+M_2+\ldots\ldots\ldots+M_n$.

Q.3. (a) LetX, Y be Banach Spaces and U , V be open sub sets of X and Y respectively. Let 'f' be a homeomorphism of $U\ onto\ V$ and g the inverse homeomorphism of X onto Y, then prove that g is differentiable at $b = f(a) \in V$.

(b) Let X and Y be Banach Space and f be a continuous function of an open subset U of X into Y. If a continuous linear map g of X into Y is a derivative of f at a point x in u then show that for each vector a in x, $Da\, f(x) = g(a) = Df(x)a$, where $D_a f(x)$ is the derivative of f at x along a vector a in x.

Q.4 (a) If [a,b] be a compact interval of R , let f be a continuous function on [a,b] into a Banach Space X and g be a continuous function on [a,b] into R, such that f and g are differentiable at each point $t \in (a, b)$ and $\|Df(t)\| \leq Dg(t)$. Then prove that:

$$\|f(b) - f(a)\| \leq g(b) - g(a)$$

Q.5(a) Let X, Y, Z be Banach Spaces and $f: X \text{ x } Y \to Z$ be a continuous bilinear function, then prove that f is differentiable and its derivative at the point $(u, v) \in X \text{ x } Y$ is given by

$$Df(u, v).(x, y) = f(x, v) + f(u, y)$$

(b) Let X, Y, Z be Banach Spaces, and U, V be open subsets of X and Y respectively. Let $f: U \to V$ and $g: V \to Z$ be continuous mappings. If f and g are of class c^n on U and V respectively, then prove that $h = gof$ is a class of c^n on U into Z.

Q.6 Let f be a continuous function on an open subset U of a Banach Space X into a Banach Space Y. If 'f' is twice differentiable at the point $a \in U$, then prove that

$D^2 f(a)\ (h, k) = D^2 f(a)\ (k, h)$ for all $(h, k) \in x^2$

Q.7 (a) State and prove Implicit Function Theorem.

(b) Let $f: [a, b] \to X$. If 'f ' is the limit of a uniformly convergent sequence of step – functions then prove that f is regulated.

Q.8 (a) If a function 'f ' on a compact interval $[a, b]$ of R into a Banach Space X is regulated then show that it has one sided limits at every point of [a,b]. That is, it has left limit at every point of [a, b] and right limit at every point of [a, b].

(b) Let I = [a, b] be a compact interval in R and X, Y be real Banach Space. Let U be an open subset of x, and 'f ' be a continuous function on $U \times I$ into Y. Then prove that a function $g: U \to Y$ defined by $g(x) = \int_a^b f(x, t)dt$ is continuous. Further, if the partial derivative D_1f with respect to the first variable exists and is continuous on $U \times I$ into $\mathsf{L}(x, y)$ then prove that g is continuously differentiable in U and $Dg(x) = \int_a^b D_1 f(x, t)dt$

M.Sc. Fourth Semester

(Assignment)

MATHEMATICS

SECOND PAPER

Viscous Fluid Dynamics-II

Attempt any two questions.

Q.1. Discuss the flow due to a plane wall suddenly set in motion in its own plane in an infinite mass of viscous incompressible fluid, which is otherwise at rest.

Q.2. Viscous incompressible fluid occupies the region $y > 0$ on one side of an infinite plate $y = 0$. The plate oscillates with a velocity $V_0\ Cos\ nt$ in the x – direction. Show that the velocity distribution of the fluid motion is given by $u = U_0 e^{-n}\ Cos(nt - \eta)$ where $\eta = \left(\frac{n}{22}\right)^{1/2} y$.

Q.3. Find out the temperature distribution in the Plane Coutte flow when the upper plate is moving in its own plane with a velocity U. The upper plate is at a temperature T_1 and the stationary plate at a temperature T_0 $(T_1 > T_0)$.

Q.4. Discuss the temperature distribution for the flow between two concentric rotating cylinders.

Q.5. Obtain expressions for velocity components and drag coefficient in the stokes flow past a sphere.

Q.6. Why Oseen equations are improvement over Stoke's equations? Obtain Oseen equations for the flow past a fixed sphere.

Q.7. Discuss the Boundary layer flow on a flat plate (Blasius – Topfer solution) and obtain the following equation $2\,\Phi'''(\eta) + \Phi(\eta)\Phi''(\eta) = 0$ with corresponding boundary conditions.

Q.8. Find a relation between heat flux and the skin friction in the solution of the thermal boundary layer equation for $P_r = 1$.

M.Sc. Fourth Semester

(Assignment)

MATHEMATICS

Third Paper

Mathematical Programming-II

Attempt any two questions.

Q.1. (a) Show that the function $1/x$ is strictly convex for $x > 0$ and strictly concave for $x < 0$.

(b) Solve the following non – linear programming problem using the Lagrange's multiplier method.

Optimize $z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$

subject to $x_1 + x_2 + x_3 = 15$

$2x_1 - x_2 + 2x_3 = 20$

and $x_1, x_2, x_3 \geq 0$

Q.2. (a) Solve graphically the following problem

Maximize $z = x_1^2 + x_2^2$

subject to $x_1 + x_2 \geq 4$

$2x_1 + x_2 \geq 5$

and $x_1, x_2 \geq 0$

(b) Determine the sign of definiteness for the matrix

$$\begin{bmatrix} 3 & 1 & 2 \\ 1 & 5 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

and find corresponding quadratic form.

Q.3. Use the Kuhn – Tucker conditions to solve the following non – linear programming problem.

Maximize $z = 2x_1 - x_1^2 + x_2$

s. t. $2x_1 + 3x_2 \leq 6$

$2x_1 + x_2 \leq 4$

and $x_1, x_2 \geq 0$

Q.4. State and prove Kuhn – Tucker necessary and sufficient conditions in non – linear programming.

Q.5. Use Wolfel's method to solve the quadratic programming problem

Max $\quad z = 2x_1 + 3x_2 - 2x_1^2$

s. t. $\quad x_1 + 4x_2 \leq 4$

$x_1 + 2x_2 \leq 2$

and $\quad x_1, x_2 \geq 0$

Q.6. Discuss duality in the quadratic programming.

Q.7. Solve the following quadratic programming problem by Beale's method:

Max $\quad z = 10x_1 + 25x_2 + 10x_1^2 - x_2^2 - 4x_1x_2$

s.t $\quad x_1 + 2x_2 + x_3 = 10$

$x_1 + x_2 + x_4 = 9$ and

$x_1, x_2, x_3, x_4 \geq 0$

When $n = k + 1$, solve the problem:

Q.8. Minimize $z = 7x_1x_2^{-1} + 3x_2x_3^{-2} + 5x_1^{-3}x_2x_3 + x_1x_2x_3, \quad x_1, x_2, x_3 \geq 0$

by geometric programming method.

M.Sc. Fourth Semester

(Assignment)

MATHEMATICS

FOURTH PAPER

Integral Equations

Attempt any two questions.

Q.1. (a) Show that the function $g(x) = xe^x$ is a solution of the volterra integral equation

$$g(x) = Sin\, x + 2 \int_0^x Cos\,(x-t)\; g(t)\, dt$$

(b) Convert the following BVP into integral equation

$$\frac{d^2y}{dx^2} + \lambda y = 0; \quad y(0) = 0, y(l) = 0$$

Q.2. (a) Convert the following differential equation into into integral equation

$$\frac{d^2y}{dx^2} + y = 0; \text{ When } y(0) = 0, \qquad y'(a) = 0$$

(b) Show that the homogenous integral equation

$$g(x) = \lambda \int_0^1 \left(t\sqrt{x} - x\sqrt{t}\right) g(t) dt$$

does not have real eigen values and eigen functions.

Q.3. (a) Solve the following integral equations by the method of successive approximations:

$$g(x) = \frac{5x}{6} + \frac{1}{2}\int_0^1 xt\; g(t)dt$$

(b) Determine the resolvent kernels for the following kernel $K(x,t) = e^{x+t};\; a = 0,\; b = 1$

Q.4. (a) Find the Resolvent Kernel of the following Volterra Kernel

$$K(x,t) = e^{x-t}$$

(b) Solve the integral equation by using the method of successive approximation

$$g(x) = x - \int_0^x (x-t)\; g(t)dt\;, \qquad g_0(x) = 0$$

Q.5. (a) Find the Eigen values and Eigen functions of the integral equation

$$g(x) = \lambda \int_{-1}^{1} (x+t)\; g(t)dt$$

(b) Prove that if $K(x,t)$ is real symmetric and continuous and $K(x,t) \neq 0$, then all the characteristics constants are real.

Q.6. (a) Prove that the Eigen functions of a symmetric kernels corresponding to distinct Eigen values are orthogonal.

(b) Prove that the set of Eigen values of second iterated kernel coincide with the set of squares of the Eigen values of the given kernel.

Q.7. (a) Using the Fredholm determinants, find the Resolvent kernel of the kernel

$K(x,t) = 1 + 3xt, 0 \leq x \leq 1, 0 \leq t \leq 1$

(b) Solve the Abel integral equation

$$f(x) = \int_0^x \frac{g(t)}{(x-t)^\alpha} dt \qquad 0 < \alpha < 1$$

Q.8 (a) Solve the integral equation

$$g(x) = 1 + \int_0^x \sin(x-t)\, g(t)\, dt$$

and verify your answer

(a) Solve for $f(x)$ the integral equation

$$\int_0^\infty f(x) \cos px \; dx = \begin{cases} 1-p & 0 \leq p \leq 1 \\ 0 & p > 1 \end{cases}$$

hence deduce that $\int_0^\infty (sin^2 t/t^2)\, dt = \Pi/2$

M.Sc. Fourth Semester
(Assignment)
MATHEMATICS
FIFTH PAPER
General Relativity & Cosmology

Attempt any two questions.

Q.1. Derive Einstein's Field Equation for Matter and Empty space.

Q.2. Explain Clock Paradox and how it is removed in General Relativity.

Q.3. Obtain Schwarzschild Exterior Metric in isotropic form. Prove that mass of the sun in gravitational unit is 1.47 km approx.

Q.4. Obtain Relativistic Differential Equation for the orbit of a planet round the sun.

Q.5. Define Advance of Perihelion for a planet. Obtain the expression for advance of perihelion for the planet Mercury.

Q.6. Derive Schwarzschild metric for a spherically symmetric ball of fluid at rest with its center at origin.

Q.7. Derive the expression for Lorentz Force on charged particle. Show that $\vec{E}^2 - \vec{H}^2$ and $\vec{E}.\vec{H}$ is Lorentz invariant.

Q.8. Obtain non-static form of De-sitter Line Element. Define Red Shift. Obtain the expression for Doppler Effect (Red Shift) using non-static form of De-sitter line element.

M.Sc. Fourth Semester

(Assignment)

MATHEMATICS

SIXTH PAPER

Numerical Analysis-II

Attempt any two questions.

Q.1 (a) Obtain normal equations for fitting a straight line to given data, using least squares principal.

(b) Using the method of least-squares, find an equation of the form

$$y = ax + bx^2$$

that fits the following data :

X	1	2	3	4	5	6
Y	2.6	5.4	8.7	12.1	16	20.2

Q.2. (a) Fit a second degree parabola to the given data:

X:	1929	1930	1931	1932	1933	1934	1935	1936	1937
Y:	352	356	357	358	360	361	361	360	359

(b) Using the Chebyshev polynomials, obtain the least squares approximation of second degree for $f(x) = x^3 + x^2 + 3$ on the interval $[-1, 1]$.

Q.3 (a) Consider the IVP :

$$y''' + 3y'' + y' + 3y = sin2t\ ,\ t \in [0,1] \quad y\,(0) = 0\ ,\ y'(0) = 1, y''(0) = 2$$

Approximate $y\,(1)$, $y'(1)$, $y''(1)$ using second order Taylor series method taking step length $h = 1$.

(b) Use Picard's method to approximate the value of y , where y satisfies the differential equation $\frac{dy}{dx} = 1 + xy$, $y(2) = 0$

Q.4. (a) Using Taylor's series for y(t) find y(2.1) correct to four decimal places if y(t) satisfies $\frac{dy}{dt} = 1 - \frac{y}{t}$ and y(2) = 2.

(b) Using fourth order Runge – Kutta method with one step, compute y(0.1) to five places of decimal if: $y' = 0.31 + 0.25y + 0.3t^2$ and $y = 0.72$ when $t = 0$.

Q.5 (a) Derive the Stability of Multi-Step Methods.

(b) Solve the Boundary Value Problem

$\frac{d^2y}{dt^2} = y, y(0) = 0, y(1) = 1$ by Shooting Method.

Q.6 Solve the second order BVP by Shooting Method :

$y'' = \left(1 - \frac{t}{5}\right) y + t\ ,\ y(1) = 2\ , y(3) = -1$

Q.7. (a) Derive the Numerov method of the order four to solve the linear boundary value problem of the type $y'' = f(x, y)$.

(b) Solve the Boundary Value Problem

$y^{iv} = 1$

$y(0) = y'(0) = y(1) = y'(1) = 0$ with $h = \frac{1}{4}$, using second order difference method.

Q.8. (a) Discuss finite difference scheme for a linear boundary value problem.

(b) Solve the boundary value problem $\frac{d^2y}{dx^2} - 64y + 10 = 0,\ y(0) = y(1) = 0$ by the finite difference method, compute y(0.5) and compare it with exact value.

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